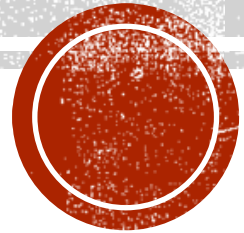
THREE PHASE SYSTEMS AND CIRCUITS

Analyze Three phase balanced and unbalanced circuits



OBJECTIVES

- State The Advantages Of Three-phase Over Single-phase System
- Explain How Three-phase Voltages Are Generated
- Distinguish Between A Balanced Supply And Unbalanced Supply System
- Distinguish Between A Balanced Load And Unbalanced Load.
- Write The Relationship Between Ina Star- And Delta- Phase, and Line Quantities Connected System
- Distinguish Between Active Power And Reactive Power .
- Calculate Current, Power, And Power Factor Of Balanced Three-phase Loads.
- Make Connection Diagram And Measure Power And Power Factor In A Three Phase -Balanced Load And Unbalanced Load
- Convert A Star-connected Into An Equivalent Delta Connected Load
- Solve Numerical Problems Of Unbalanced-connected System. Star- And Delta- Connected Loads.

1.INTRODUCTION

- Generation, Transmission And Distribution Of Electricity Is Done By Three-phase Electrical Net-works. Consisting Of Generators, Transformers And Transmission And Distribution Lines Forming the Power System.

- In A Three-phase System, We Have Three Independent Voltages Induced In The three Windings Of The Generator. To Understand The Difference Between A Single-phase Voltage and A Three-phase Voltage System, Let Us Consider **How These Voltages Are Generated In AC Generators,** We Have Known That EMF Is Induced In A Coil If It Cuts Lines Of Force, In Figure, We have Placed One Coil In Slots Of A Hollow Cylindrical Stator Core.

- We Have Seen That The Coil Is Stationary And The Magnetic Field Is Being Rotated. However, We Could Have Had The Field Stationary And The Coil Rotating By Placing The Field Magnets On The Stator And The Coil On A Cylindrical Rotor.

- What Is Required Is To Produce A Relative motion Between Magnetic Field And Conductor. Magnitude Of Induced EMF Will depend Upon No. Of Coils, Strength Of Magnetic Field And Speed Of Rotation Of The Magnet.

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

1.The Frequency Of The Induced EMF Will Depend upon The Number Of Magnetic Poles confronted By The Coils Per Revolution. Normally, The Number Of Magnetic Poles And The Speed of Rotation Of The Magnetic Poles By A Drive, Usually A Turbine, Are Kept Constant,.

2 In A Three-phase System, We Will Have Three-phase Voltages Induced In The Three-phase Windings Of The Generator. In All Generating Stations, Three-phase Generators Are Installed.

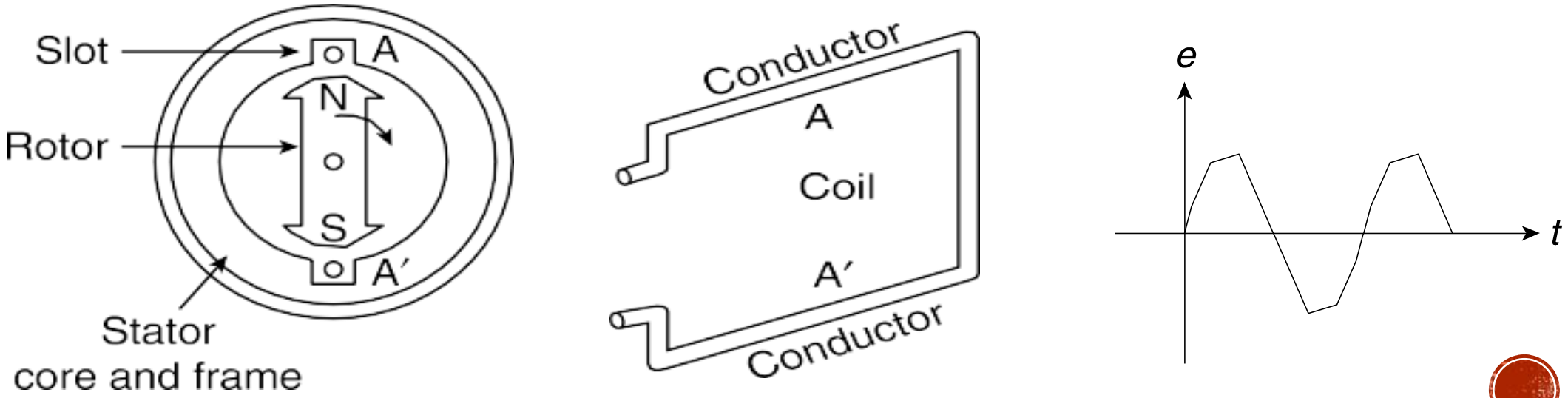


Fig. 1 generation of Single-phase Voltage

2. ADVANTAGES OF THREE-PHASE SYSTEMS

- Three-phase System Has A Number Of Advantages Over A Single-phase System.

1. The Output Of A Three-phase Machine Generating Electricity Is **More** Than The Output Of A single-phase Machine Of The Same Size,

2. The Most Commonly Used Three-phase Induction Motors **Are Self Starting**. For Single-phase motors, A Separate Starting Winding Is Required.

3. Electrical Power Transmission From The Generating Station To The Places Of Use Is Done By transmission Lines, It Has Been Seen That Three-phase Power Transmission Is **More Economical** Than Single-phase Power Transmission.

4. The Power Factor Of Three-phase Systems Is **Better Than** That Of The Single-phase Systems.

5. Single-phase Supply Can Also Be **Obtained** From A Three-phase Supply,

6. The Instantaneous Power In A Single-phase System Is Fluctuating With Time Giving Rise To **noisy Performance** Of Single-phase Motors. The Power Output Of A Symmetrical Three-phase system Is Steady,

7. For Rectification Of AC Into DC, The DC Output Voltage Becomes **Less Fluctuating** If The nimbler Of Phases Is Increased,

Thus, We See That From Generation, Transmission, Distribution And Utilization Points Of View, three-phase Systems Are Preferred Over Single-phase Systems Due To A Number Of Reasons Mentioned Above**.**

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

3. GENERATION OF THREE-PHASE VOLTAGES

- Due To A Number Of Practical Considerations, Generators Are Built To Generate Poly-phase Voltages. Commercial Generators Are Built To Generate Three-phase Voltages.
- In Three-phase Generators, Three Separate Windings Are Made. Windings Are Made Of Coils. These Windings Are Placed In Stator Slots At An Angle Of **120°** Apart, As Shown In **Fig .2** **RR'** Is one Phase Winding, **YY'** Is The Second Phase Winding And **BB'** Is The Third Phase Winding,
- The Three-phase Windings Are Placed At An Angular Distance Of 120°, For Simplicity, Only One Coil Per Phase Has Been Shown. In Practice, A Number Of Coils Connected In Series Make One Phase Winding.
- There Will Be A Time Phase Difference Between The Emfs Induced In These Coils (Windings). The Time Phase Difference Will Be 120°. In Terms Of Time, The Phase Difference Will Be The Time Taken By The Magnetic Poles To Rotate By 120°, That Is, One-third Of A Revolution. Thus, Across The Three-phase Windings, We Will Get Three Voltages That Are Equal In Magnitude And Frequency But Having A Time Phase Difference Of 120° Between Them, As Shown In **Fig .3**

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

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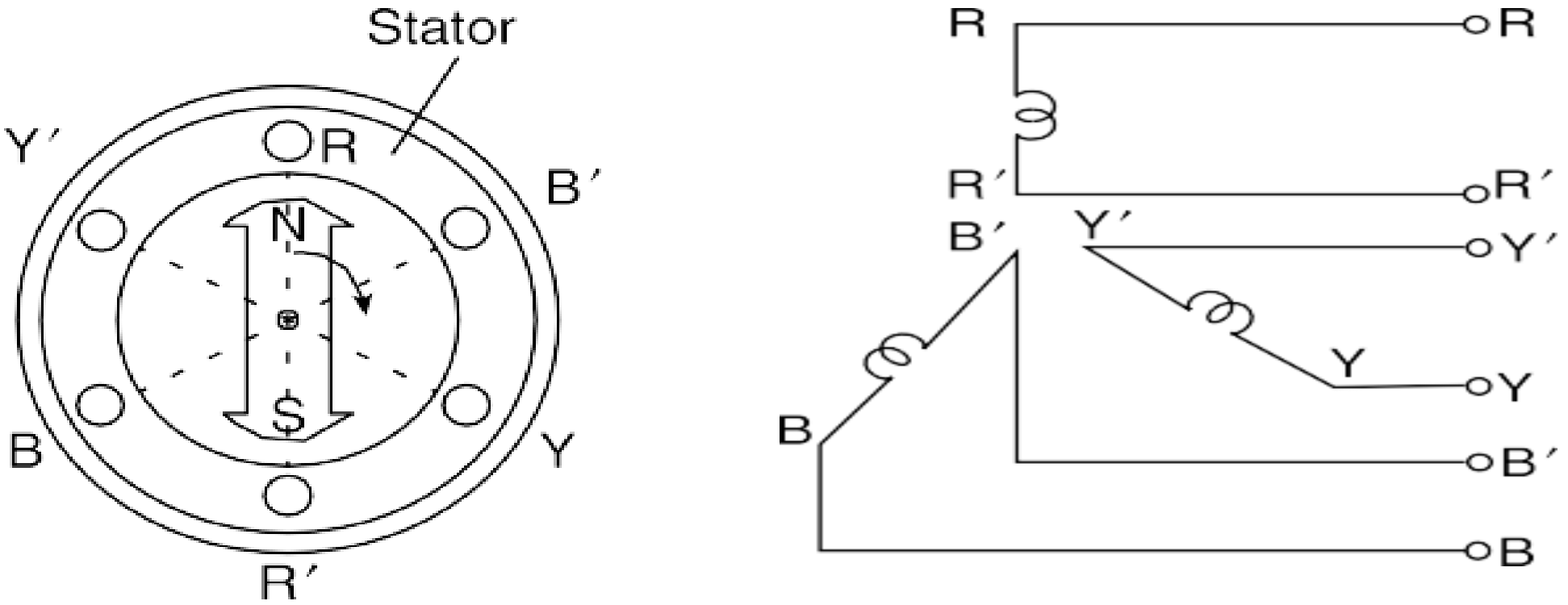


Fig. 2 Generation of Three-phase Voltages in the Three-phase Windings

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

3.1 Equation Of Three-phase Voltages

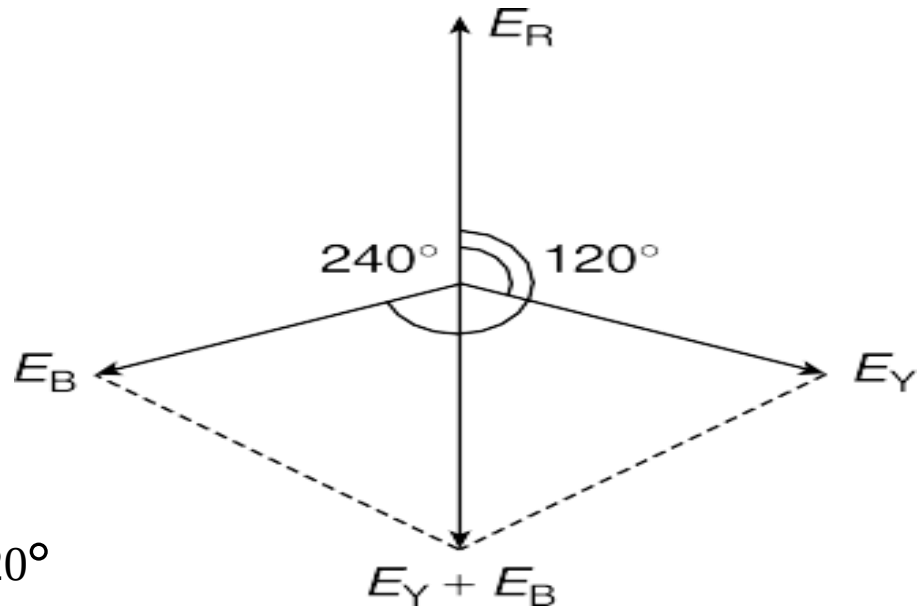
Equation Of Three-phase Voltages the Equations Of Voltages Are As Follows:

eR =Em Sin ωt ----------1, eY = Em Sin (ωt — 120°) ------2, eB = Em Sin (ωt— 240°) ------- 3

Resultant EMF = Em Sin ωt+ Em Sin (ωt — 120°)+Em Sin (ωt— 240°) = 0

- Since The Three Voltages Are Equal In Magnitude But Displaced In Time Phase By 120°, Their Phasor sum Is Zero as Shown In **Fig .3.**

- Three-phase Supply Is Required For Large-capacity Electrical loads, These Loads Could Be Three-phase Motors Used In Indus trial, Commercial, Agricultural And Other Sectors, For Example, the Water Pump Used For Irrigation Purpose Is Invariably A **Fig .3**



Three phase Voltages Displaced In Time Phase By 120°

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

- Three-phase Motor-driven Pump Requiring A Three-phase Supply. Therefore, Similar To The Three-phase Supply, We Will Have Three-phase Loads. Three-phase Supply Will Be Supplying Electrical Power To Three-phase Loads.
- A Number Of Terms Are Used In Connection With Three-phase Supply And Three-phase Loads. These Are Described As Follows. Further, The Three-phase Windings Can Be Connected Together In The Form Of **Star Or Delta**.
- The Voltage Between The Two-phase Windings And Current Flowing Through The Phase Windings, And The Supply Line Will Be Different In Star And Delta Connections. These Will Be Studied In Detail.
- By Now, We Must Have Realized That By Phase We Mean A Winding. A Phase Difference Between Two Windings Is The Physical Angular Displacement Between Them. In A Three-phase Winding, The Phase Difference Between The Windings Is **120°.** Phase Sequence Is The Order In Which Maximum Voltage Is Induced In The Windings.
- If The Magnetic System Of Figure .2 Rotates In The Anticlockwise Direction, The Phase Sequence Of EMF Induced In The Three Phases Will Be RBY.

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

ELEMENTARY THREE-PHASE GENERATOR

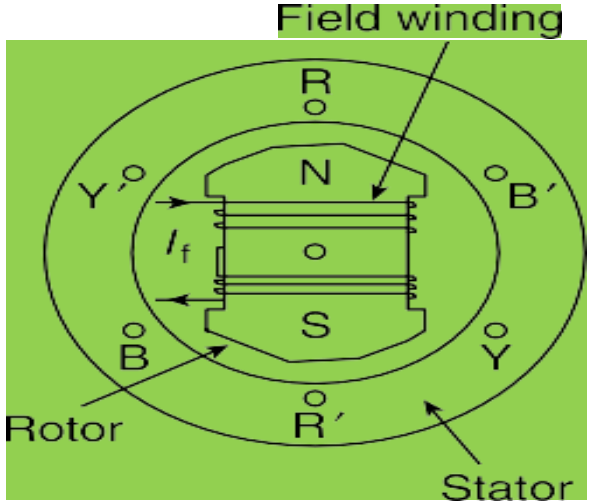
- Let Us Consider An Elementary Three-phase Generator As Shown In Fig .4(a), Three phase windings Are Placed In Slots In The Stator. For Simplicity, Only One Coil Per Phase Has Been Used. R-R' Is One Coil Making R-phase Windings. Y-Y' Is Another Coil Forming Y-Phase Winding, BB' Is one Col Forming B-Phase Winding. These Three-phase Windings Are Placed In The Stator Slots At An angle Of 120° In Space.

- For Simplicity, Only One Coil Per Phase Has Been Shown. In Actual Practice, a Number Of Coils Are Connected Together To Form Each Phase Winding. The Rotor Carries The Magnetic Poles That Produce The Magnetic Field.

- Direct Current Is Supplied To The Field Windings So That the Field Magnets Are Excited. When The Rotor Is Rotated By A Turbine. The Magnetic Flux Are Cut By the Stator Coils RR', YY' And BB' In Sequence.

- The Emfs Induced In These Coils Are Sinusoidal In nature Because Of The Nature Of Flux Distribution. The Voltage Induced In The Three-phase Windings. Will Be Identical In Nature But They Will Be Displaced In Time-phase By 120° As Has Been Shown..

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh



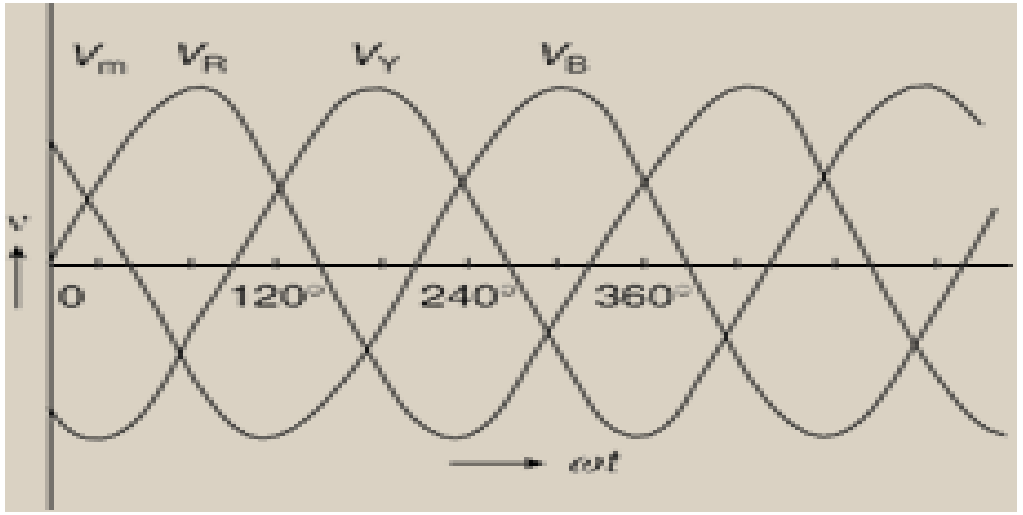


Fig.4 Generation Of Three phase Voltages
(a) Three phase Two-pole Generator and (b) Three phase Voltages Induced In The Windings

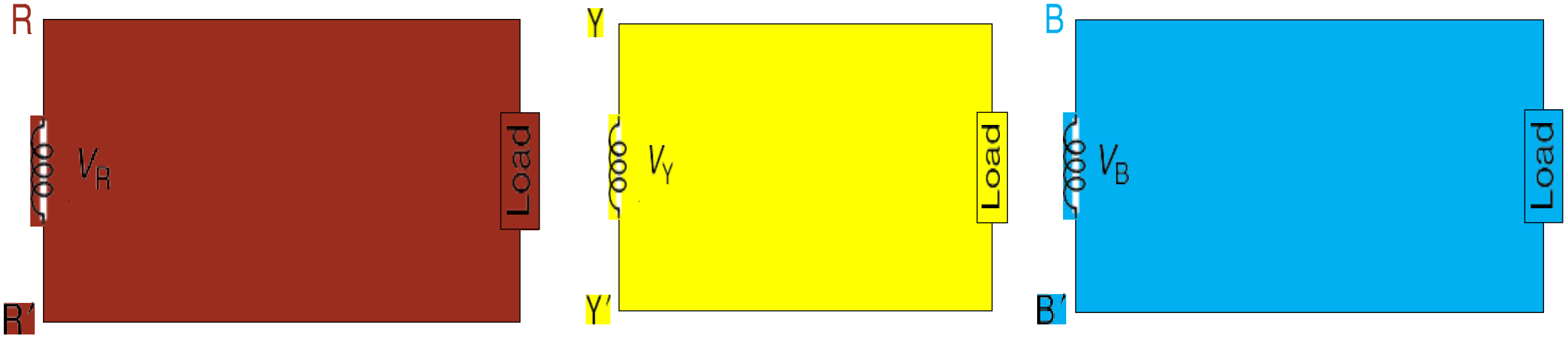


Fig. 5 the three-phase Windings are Connected to the Load Independently Through Six Wires

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

3.2 Balanced Three- phase System

- In A Balanced Three-phase Supply System All The Phase Voltage Are Equal In Magnitude But Displaced By 120°. A Balanced Star-connected Supply System Is Shown In **Fig. 6**, The Phase Voltage And The Line Voltages Have Been Represented Through Phases In The Figure,

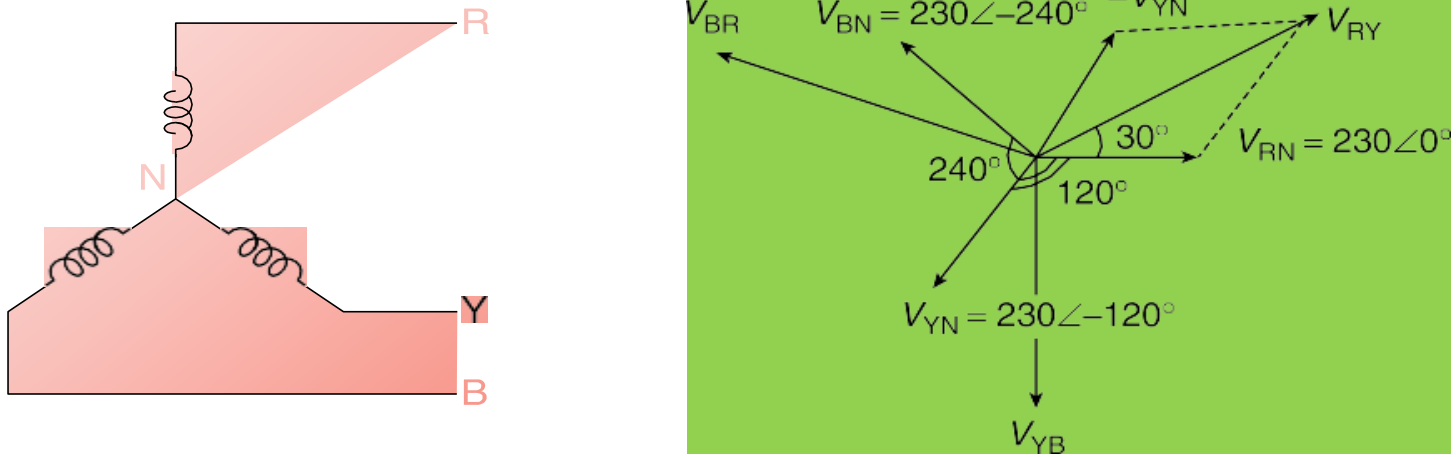


Fig .6 Representation of a Balanced Star-connected Supply System

Here, Let $\mathbf{V_{RN} = 230\angle o^\circ}$ **then** $\mathbf{V_{YN} = 230\angle -120^\circ}$ $\mathbf{V_{BN} = 230\angle -240^\circ}$

- the Corresponding Line Voltages Are equal To $\sqrt{3}$ Times The Phase Voltage And Leading The Corresponding Phase Voltage By 30°.
- We Draw The Line Voltage By Assuming A Phase Sequence Of **RYB**.
  - $\mathbf{V_{RY} = V_{RN} - V_{YN} = \sqrt{3} * 230\angle +30^\circ = 400\angle 30}$
  - $\mathbf{V_{YB} = V_{YN} - V_{BN} = \sqrt{3} * 230\angle -90^\circ = 400\angle -90\ deg}$
  - $\mathbf{V_{BR} = V_{BN} - V_{RN} = \sqrt{3} * 230.\angle -210^\circ = 400.\angle -210^\circ}$
- like Phase Voltages, The Line Voltages Have Also A Phase Difference Of 120°.

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

4. TERMS USED IN THREE-PHASE SYSTEMS AND CIRCUITS

- The Following Are Some Of The Terms Used While Describing A Three-phase System. These Are As Follows:

1. Balanced Supply: A Set Of Three Sinusoidal Voltages (Or Currents) That Are Equal In Magnitude But Has A Phase Difference Of 120° Constitute A Balanced Three-phase Voltage (Or Current) System..

2.Unbalanced Supply: A Three-phase System Is Said To Be Unbalanced When Either Of The Three-phase Voltages Are Unequal In Magnitude Or The Phase Angle Between The Three Phases Is Not Equal To 20°,bahae.

3. Balanced Load: If The Load Impedances Of The Three Phases Are Identical In Magnitude As Well As Phase Angle, Then The Load Is Said To Be Balanced. It Implies That The Load Has The Same Value Of Resistance A And Reactance A) And / Or A,. In Each Phase.

4. Unbalanced Load: If The Load Impedances Of The Three Phases Are Neither Identical In Magnitude Nor In Phase Angle, Then The Load Is Said To Be Unbalanced.

5. Single Phasing: When One Phase Of The Three-phase Supply Is Not Available Then The Condition Is Called Single Phasing,.

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

6.Phase Sequence: The Order In Which The Maximum Value Of Voltages Of Each Phase Appear Is Called The Phase Sequence. It Can Be RYB Or RBY..

7.Coil: A Coil Is Made Of Conducting Wire, Say Copper, Having An Insulation Cover. A Coil Can Be Of A Single Turn Or Many Number Of Turns. Normally, A Coil Will Have A Number Of Turns. A Single Turn Of A Coil Will Have Two Conductors On Its Two Sides Called Coil Sides.&.

8. Winding: A Number Of Coils Are Used To Make One Winding. Normally, The Winding Coils Are Connected In Series. One Winding Forms One Phase.

9. Symmetrical System: In A Symmetrical Three-phase System, The Magnitude Of Three-phase Voltages Is The Same But There Isa Time Phase Difference Of 120° Between The Voltages.

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

5. THREE-PHASE WINDING CONNECTIONS

- A Three-phase Generator Will Have Three-phase Windings. These Phase Windings Can Be Connected In Two Ways:

1. **Star Connection** 2. **Delta Connection**

5.1 Star Connection: the Star Connection Is Formed By Connecting The Starting Or Finishing Ends Of all The Three Windings Together. A Fourth Conductor, That Is, Taken Out Of The Star Point Is Called The Neutral Point. the Remaining Three Ends Are Brought Out For Connection To Load. These Ends Are Generally referred To As R-Y-B, To Which The Load Is To Be Connected. The Star Connection Is Shown In **fig (a).** This Is A Three-phase, Four-wire Star-connected System. If no Neutral Conductor Is taken Out From The System It Gives Rise To A Three-phase, Three-wire Star-connected System.

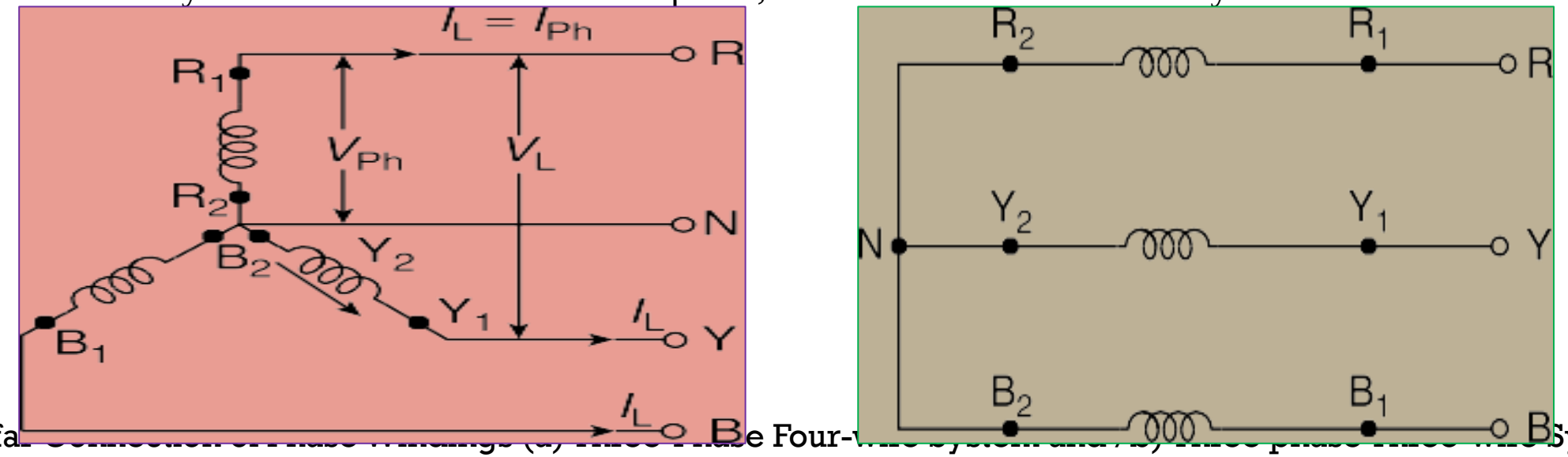


Fig. Sfa[illegible] (a) [illegible] Four-[illegible] System

5.2 DELTA CONNECTION

- The Delta Connection Is Formed By Connecting The End Of One Winding To The Starting End Of The Other And Connections Are Continued To Form A Closed Loop, In This Case, The Current Flowing Through Each Line Conductor Is Called Line Current I_L And The Current Flowing Through Each Phase Winding Is Called Phase Current I_{Ph}. However, We Find That The Phase Voltage Ts The Same As The Line Voltage In A Delta Connection. The Delta Connection Of Windings Has Been Shown In Fig.

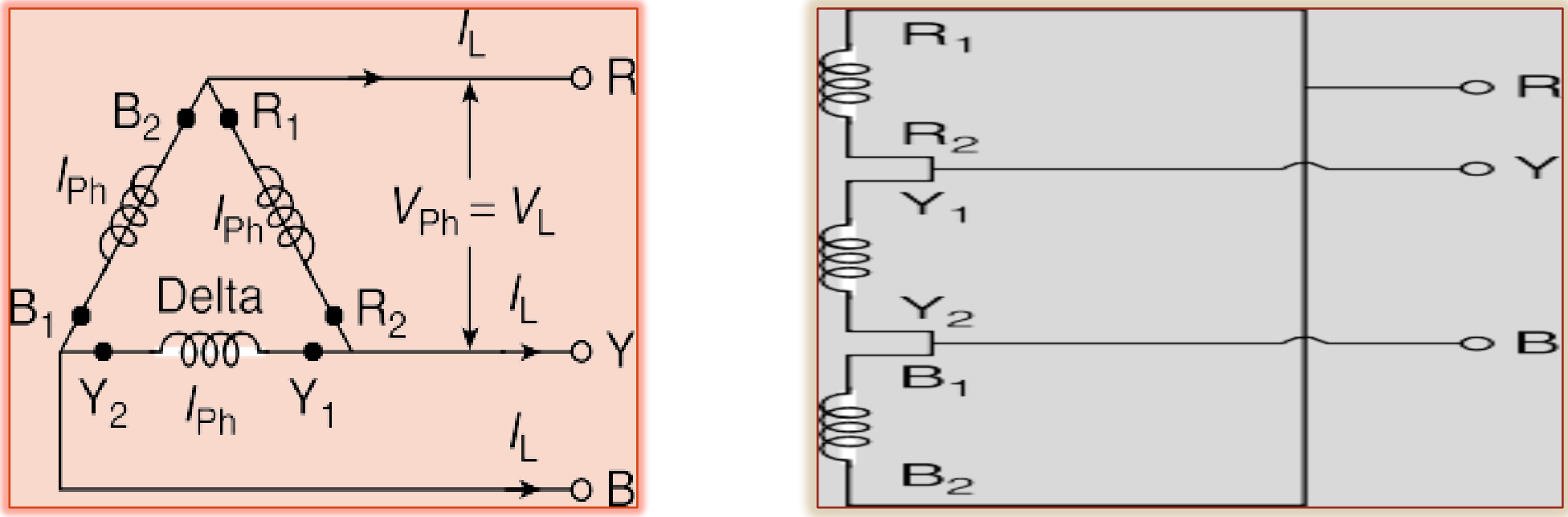
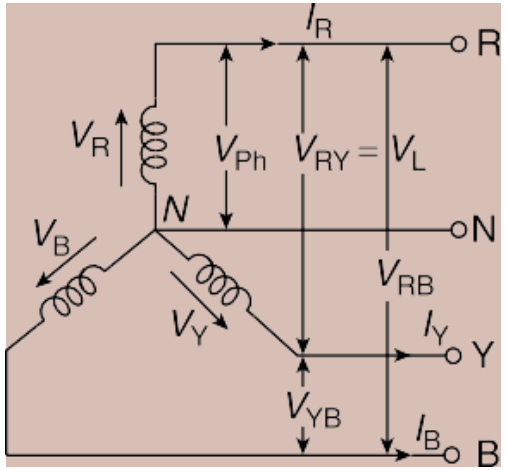


Fig. Delia Connection of Three-phase Winding's (a) Three-phase Three-wire Delta Connection and (b) Connection scheme of Windings Forming a Delta

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

5.3 RELATIONSHIP OF LINE AND PHASE VOLTAGES AND CURRENTS IN A STAR-CONNECTED SYSTEM

- Consider The Balanced Star-connected System As Shown In Fig. suppose, The Load Is Inductive And, Therefore, Current Will Lag The Applied Voltage By An Angle & Consider A Balanced System So That The Magnitude Of Current And Voltage Of Each Phase Will Be The Same.



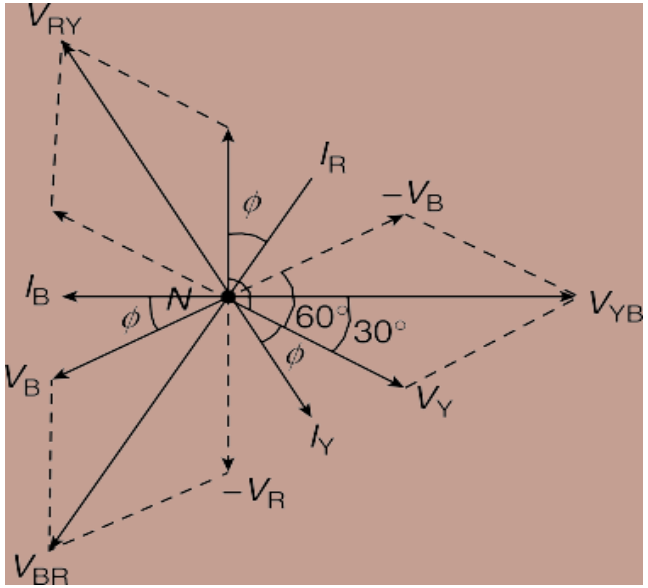
That is, phase voltages, $V_R = V_Y = V_B = V_{Ph}$

Line current $I_R = I_Y = I_B = I_L$

Line voltage $V_L = V_{RY} = V_{YB} = V_{BR}$

Phase current $I_{Ph} = I_R = I_Y = I_B$

$I_L = I_{Ph}$ for star connection as the same phase current passes through the lines to the load.



Balanced Star-connected System (a) Three-phase Sypply System and (b) Phasor Diagram

To derive the relation between V_L and V_{Ph}, consider line voltage V_{RY}.

$$V_{RY} = V_{RN} + V_{NY}$$

$$V_{RY} = V_{RN} + (-V_{YN})$$

Similarly, $V_{YB} = V_{YN} + (-V_{BN})$

and $V_{BR} = V_{BN} + (-V_{RN})$

- The Procedure For Drawing The Phasor Diagram Of Fig (b) Is As Follows.

Draw Three Phasors V_R, V_Y And V_B, Representing The Phase Voltages. These Voltages Are Of equal Magnitude But Displaced By 120°. The Line Voltage Phasors V_{RY}, V_{YB}, And V_{BR}, Are Drawn by Vectorially Adding The Phase Voltages. For Example, To Draw Line Voltage V_{RY}. We Have To Add the Phase Voltages As Follows:

$$V_{RY}=V_{RN}+V_{NY}=V_{RN}+(-V_{YN})$$

The phasor $V_{YN.}$ is obtained by reversing $V_{NY,}$ V_{RY}, is obtained by vectorially adding V_{RN} and V_{YN} Shown in Fig.(b). Similarly, the other line voltages have been drawn. The phase currents I_R, I_Y, and I_B, have been shown lagging the phase voltages by the power factor angle ø

From the phasor diagram shown in Fig (b), the phase angle between phasors V_R, and $(-V_Y)$is 60°.

$$V_{RY}=\sqrt{V_R^2+V_Y^2+2V_RV_Y\cos 60^\circ}$$

$$V_{RY}=V_L=\sqrt{V_{Ph}^2+V_{Ph}^2+2V_{Ph}V_{Ph}\times\frac{1}{2}}$$

$$V_L=\sqrt{3}V_{Ph} \qquad \text{---------4}$$

Thus, for the star-connected system, the following can be stated:

Line voltage = √3 x phase voltage

Line Current = phase current

Power: Power output per phase = Vph *Iph *cosø

Total power output = 3Vph Iph cosø = $3*\frac{V_L}{\sqrt{3}}*I_L$ cosø.

$$P=\sqrt{3}\,V_L\,I_L\cos\text{ø} \qquad \text{----------5}$$

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

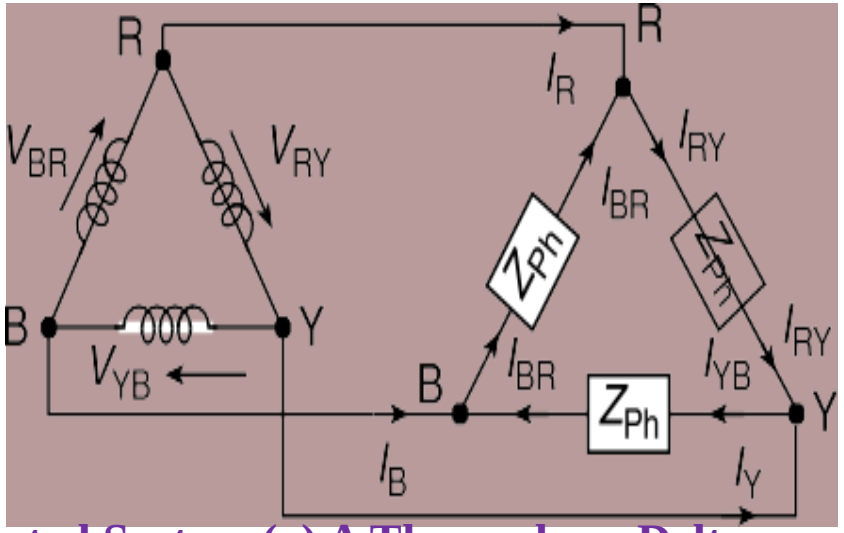
5.4 RELATIONSHIP OF LINE AND PHASE VOLTAGES AND CURRENTS IN A DELTA-CONNECTED SYSTEM

- Consider The Balanced Delta-connected System As Shown In Fig.In a Delta-connected System, The Voltage Across The Winding, That Is, The Phases Is The Same as That Across The Line Terminals. However, The Current Through The Phases Is Not The Same As through The Supply Lines, therefore, In The Case Of The Delta-connected Circuit, The Phase Voltage Is Equal To The Line voltage, But The Line Current Is Not Equal To The Phase Current.

Line voltage	$V_L = V_{RY} = V_{YB} = V_{BR}$
Line current	$I_L = I_R = I_Y = I_B$
Phase voltage	$V_{Ph} = V_{RY} = V_{YB} = V_{BR}$
Phase current	$I_{Ph} = I_{RY} = I_{YB} = I_{BR}$

Therefore Vph=V_L for delta-connected load.

Fig(a) shows a three-phase delta-connected supply system connected to a three-phase delta-connected load. The line currents are I_R,I_Y and I_B. The phase currents are I_{RY}, I_{YB}, and I_{BR}: The phasor diagram tm Fig(b) has been developed by first showing the three-phase



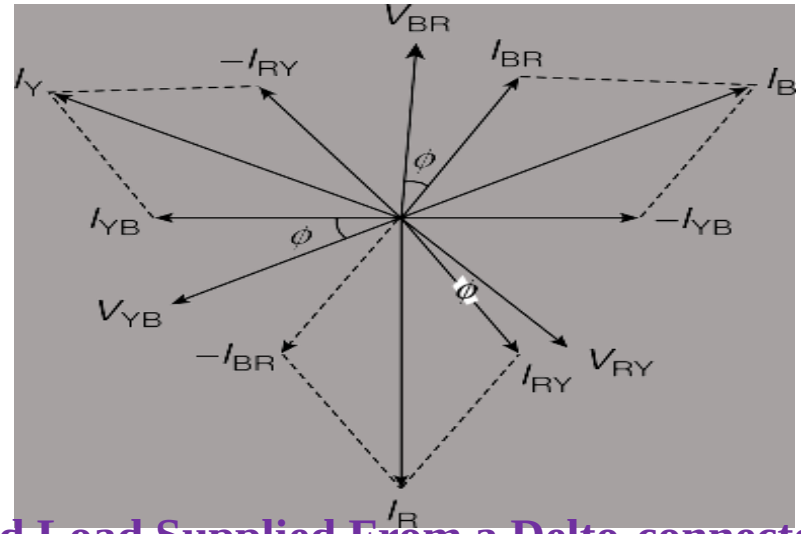


Fig Delta-connected System (a) A Three-phase Delta-connected Load Supplied From a Delto-connectéd Supply Source (b) Phasor Diagram of Voltages and Currents

voltages V_{YB},V_{BR}, and V_{RY}, of equal magnitude but displaced by 120° from each other. Then, the phase currents I_{YB},I_{BR} and I_{RY} have been shown lagging respective phase voltages by power factor angle ø. The line currents are drawn by applying KCL at the nodes R, Y, B and adding the phasors, as shown,

To derive the relation between I_L, and I_{ph}, apply KCL at node R, as shown in Fig(b)

$$I_R + I_{BR} = I_{RY}$$

Therefore $I_R = I_{RY} - I_{BR}$

Similarly, at node Y and B, we can write the following:

$$I_Y = I_{YB} - I_{RY} \qquad I_B = I_{BR} - I_{YB}$$

Since the phase angle between phase currents I_{RY}, and $-I_{BR}$ is 60°, the following can be obtained as follows:

$$\therefore I_R = \sqrt{I_{RY}^2 + I_{BR}^2 + 2 I_{RY} I_{BR} \cos 60^\circ}$$

$$I_R = I_L = \sqrt{I_{Ph}^2 + I_{Ph}^2 + 2 I_{Ph} I_{Ph} \times \frac{1}{2}} \qquad \mathbf{I_L} = \sqrt{3} \times I_{Ph}$$

Thus, for a three-phase delta-connected system, Line current = √3 x phase current

Line voltage = phase voltage

Power: power output per phase = V_{Ph} I_{ph}*cosø

Total power quiput = **3*Vph *Iph *cosø = 3*** $\frac{I_L}{\sqrt{3}} * \mathbf{V_L}$ **cosø=** $\mathbf{\sqrt{3} V_L I_L}$ **cosø**

Power = √3*line voltage*line current * power factor

or both star-connected and delta-connected systems, the total power P is

$$\mathbf{P = \sqrt{3} V_L I_L \cos\phi}$$

If per phase power is P_h, and total power is P_T, then

$$P_T = 3P_h$$

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

6. ACTIVE AND REACTIVE POWER

The expression for average power in an AC circuit is

$$P=V(I\cos\phi).$$

The quantity in brackets ts the projection of current phasor I into voltage phasor V. Accordingly in-phase component of Iph, along V has been shown in Fig.11 $I_{ph}\cos\phi$ and the perpendicular component as $I_{ph}\sin\phi$. If we multiply all the sides of the triangle ABC by Vph the triangle becomes a power triangle where AB = Vph Iph cos ø is called the active power, BC = Vph Iph sin ø is called the reactive power and Vph Iph is called apparent power.

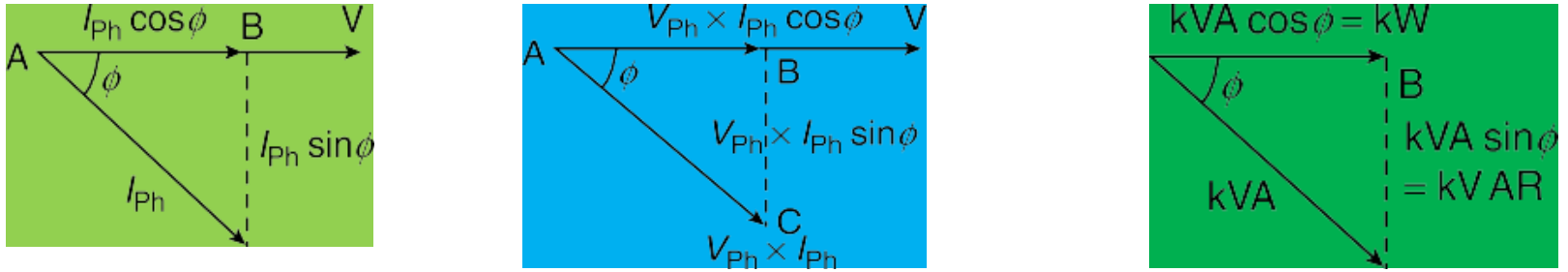


Fig.11 Power Triangle Showing the Relationship Between (a) Active Power,(b) Reactive power and (c) apparent power

Apparent power = kVA --------9

Apparent power *cos ø= Active power =kW --------10

Apparent power*cos ø= Reactive power=KVAR---11

Multiplying all the sides of the power triangle by 10^3, that is, expressing the power in termsof 'kilo^3', the power triangle is redrawn, as shown in Fig11(c) (KvA is kilo Volt Ampere).

KVA cos ø=KW (kilo-Watt)--------12

(KVA sin ø=KVAR (kilo- VAR) ------13

7. COMPARISON BETWEEN STAR CONNECTIONAND DELTA CONNECTION

As Mentioned Earlier, The Three Windings Of A Generator Can Be Connected Either In Star Or In Delta. Same Is The Case With The Transformers. Three-phase Electrical Loads And The Windings Of Three-phase Motors Can Also Be Connected In Star Or In Delta, The Relationship Between Voltages, Current And Their Phase Relationship Along With Some Other Related Factors For Star And Delta Connections Have Been Compared And Are Presented In table 1.

Star Connection	Delta Connection
1. Line current is the same as phase current, that is, $I_L = I_{Ph}$	Line current is $\sqrt{3}$ × the phase current, that is, $I_L = \sqrt{3} I_{Ph}$
2. Line voltage is $\sqrt{3}$ the phase voltage, that is, $V_L = \sqrt{3} V_{Ph}$	Line voltage is the same as phase voltage, that is, $V_L = V_{Ph}$
3. Total power = $\sqrt{3} V_L I_L \cos\phi$	Total power = $\sqrt{3} V_L I_L \cos\phi$
4. Per phase power = $V_{Ph} I_{Ph} \cos\phi$	Per phase power = $V_{Ph} I_{Ph} \cos\phi$
5. Three-phase three-wire and three-phase four-wire systems are possible	Three-phase three-wire system is possible
6. Line voltages lead the respective phase voltages by 30°	Line currents lag the respective phase currents by 30°

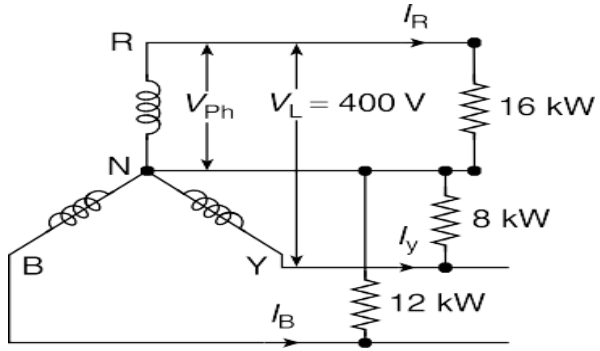
Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

EXERCISE PROBLEMS

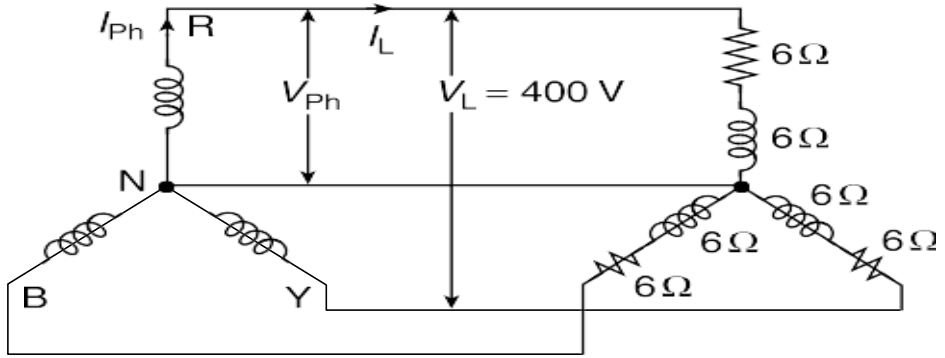
Example.1 A 400, Three-phase, 50 Hz Power Supply Is Applied Across The Three Terminals of A Delta-connected Three-phase Load. The Resistance And Reactance Of Each Phase Is 60 Andl80), Respectively. Calculate The Line Current, Phase Current, Active Power, Reactive Power And apparent Power Of The Circuit.

Example.2 A balanced star-connected load of (8 + j6)Ω per phase is connected to a balanced three-phase, 400V supply. Find the line current, power factor, power and total volt-amperes.

Example.3 A three-phase four-wire supply system has a line voltage of 400V. Three non-inductive loads of l6kW, 8KW and 12kW are connected between R, Y and B phases and the neutral, respectively, Calculate the current flowing through the neutral wire.



Example.4 A three-phase load has a resistance and reactance of 6Ω each for all the three phases. The load is connected in star. A 400 V. 50 Hz, three-phase supply is connected across the load, Calculate phase voltage, phase current, power factor, power consumed per phase and the total power consumed by the load.



Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

Example 5. A 400 V, 50:hz, Three-phase Supply Is Provided To A Three-phase Star-connected Load, Each Phase Of The Load Absorbs A Power Of 200W. The Load Power Factor Is 0.8 Lagging. Calculate The Total Power Supplied To The Load And The Line Current.

Example 6. A 400V, 50:Hz, three-phase supply is provided to a three-phase delta-connected load, The resistance and inductance of each phase of the load is 8Ω and 0.04 H, respectively. Calculate the phase current and the line current drawn by the load. Further, calculate the total power consumed.

Example 7. A three-phase star-connected load consumes a total of 12kW at a power factor of 0.8 lagging when connected to a 400M, three-phase, 50 Hz power supply. Calculate the resistance and inductance of load per phase.

Example 8. A balanced three-phase star-connected load of 8 +j6 Ω per phase is supplied by a 400 V, 50HZ supply. Calculate the line current, power factor, active and reactive power.

Example 9. A delta-connected three-phase motor load is supplied from a 400V, three-phase,S0 Hz supply system. The line current drawn is 21.A. The input power 11 kW. What will be the line current and power factor when the motor windings are delta connected?

Example 10. A balanced star-connected load of (4 +j6) Ωper phase is connected across a 400 V, three-phase, 50Hz supply. Calculate line current, phase current, line voltage, phase voltage, power factor, total power and reactive power.

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

8. MEASURMENT OF POWER IN THREE-PHASE CIRCUITS

We Have Known That In Dc Circuits, Power Is Measured As The Product Of Voltage And Current, That Is, Power

P= VI, Dc Power Can Be Measured Using A Voltmeter And An Ammeter. In AC Circuits, Power P= VI Cos ϕ In Three-phase Ac Circuits, Total Power Is Three Times The Power Per Phase.

Wattmeter Is An Instrument Used For The Measurement Of Power In AC Circuits. Wattmeter's Are Available As Single-phase Wattmeter's And Three-phase Wattmeter's. Single-phase Wattmeter's Can Be Used To Measure The Three-phase Power.

In The Case Of Star-connected Balanced Load With Neutral Connection, Only One Single-phase Wattmeter Can Be Used To Measure The Three-phase Power, The Three-phase Power Is Three Times The Single-phase Power.

For Unbalanced Three-phase Loads, That Is, Currents In The Three Phases Are Not The Same, And Hence Two Wattmeter's Are To Be Used To Measure The Three-phase Power. These Methods Are Described In The Following Sections.

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

8.1 One-wattmeter Method

In This Method, Only One Single-phase Wattmeter Can Be Used To Measure The Total Three-phase power, In This Method, The Current Coil (CC) Of The Wattmeter Is Connected In Series With Any phase And The Pressure Coil (PC) Is Connected Between That Phase And The Neutral, As Shown in Fig 16, One-wattmeter method has a de-merit that even a slight degree of unbalance in the load produces a large error in the measurement. In this balanced method, one wattmeter will measure only the power of one phase, Hence, the total power is taken as three times the wattmeter reading. Total power= 3 x Vph Iph cos ϕ.

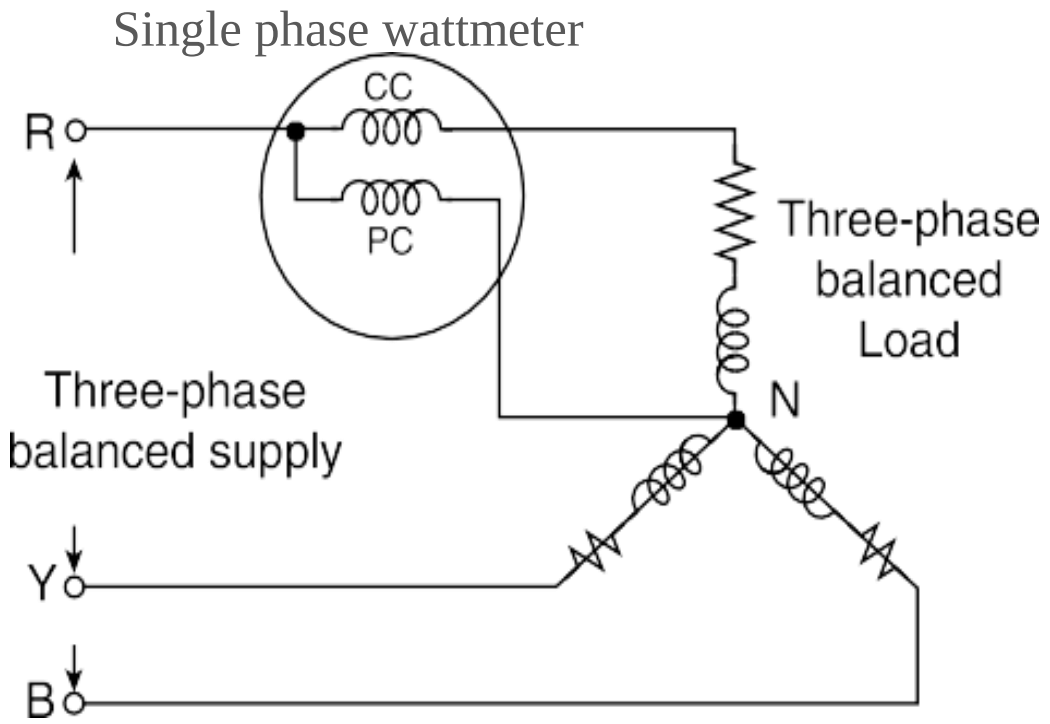


Fig.16 one-wattmeter Method of Measuring Power of a Sfar-connected Balanced Three-phase Load

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

8.2 TWO-WATTMETER METHOD

- This Method Requires Only Two Wattmeter's To Measure Three-phase Load For Balanced As Well As Unbalanced Loads. In This Method, Two Wattmeter's Are Connected In Two Phases And Their Pressure Coils Are Connected To The Remaining Third Phase, As Shown In Fig.17.

- This method of measurement is useful for balanced and unbalanced loads. Let us consider the measurement of three-phase power of a star-connected load using two single-phase wattmeter's as shown in Fig.18(a). We will calculate the power measured by the two-wattmeter's separately. Let $W_1$and W_2, respectively, be the two-wattmeter readings. The current flowing through the current coil of wattmeter W_1 is I_R. The voltage appearing across its pressure coil is V_{RB}. The wattmeter reading will be equal to **$W1=V_{RB}\ I_R$ cos of angle between V_{RB} and I_R**. Similarly, the wattmeter reading W_2, will be equal to **$W_2= V_{YB}\ I_B$ cos of angle between $V_{YB,}$ and I_B**. We will now draw the phasor diagram, and calculate W_1, and W_2From the phasor diagram, as shown in Fig.18(b) we get the equation as follows:

$$W_1\text{-}V_{RB}\ I_L \cos(30\text{-}\phi) = \sqrt{3}\ V_{ph} \cos(30\text{-}\phi) = V_L\ I_L \cos(30\text{-}\phi)$$

- **Further, $W_2=V_{YB}\ I_Y \cos(30+\phi) = \sqrt{3}\ V_{ph} \cos(30+\phi) = V_L\ I_L \cos(30+\phi)$**
- We know that the **total power in a three-phase circuit is $3\ V_{ph}\ I_{ph}\cos\phi$ or equal to $\sqrt{3}\ V_L\ I_L\ \cos\phi$.**

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

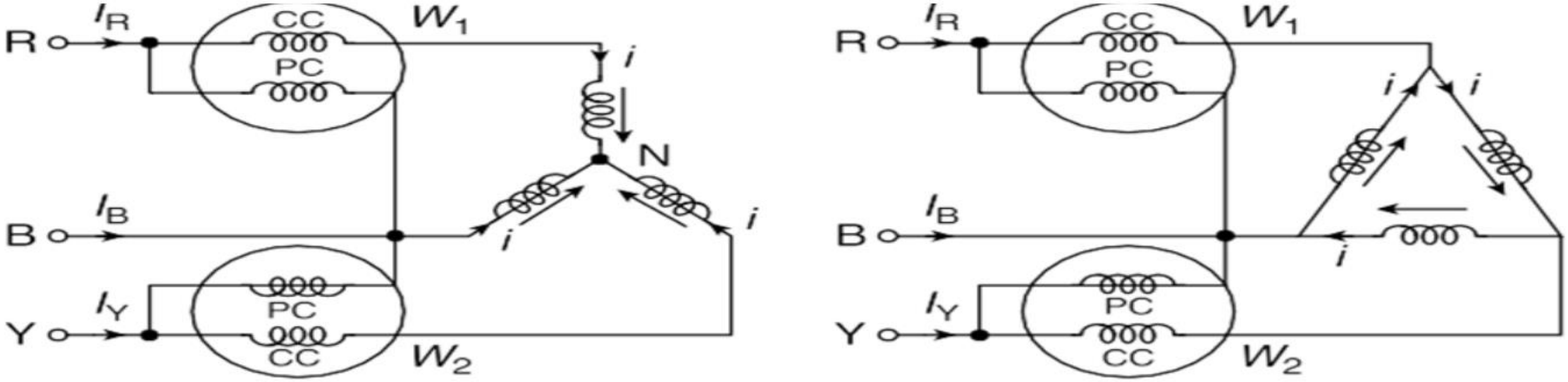


Fig 17 Two-wattmeter Method of Measuring Power for Star- and Delia-connected Load

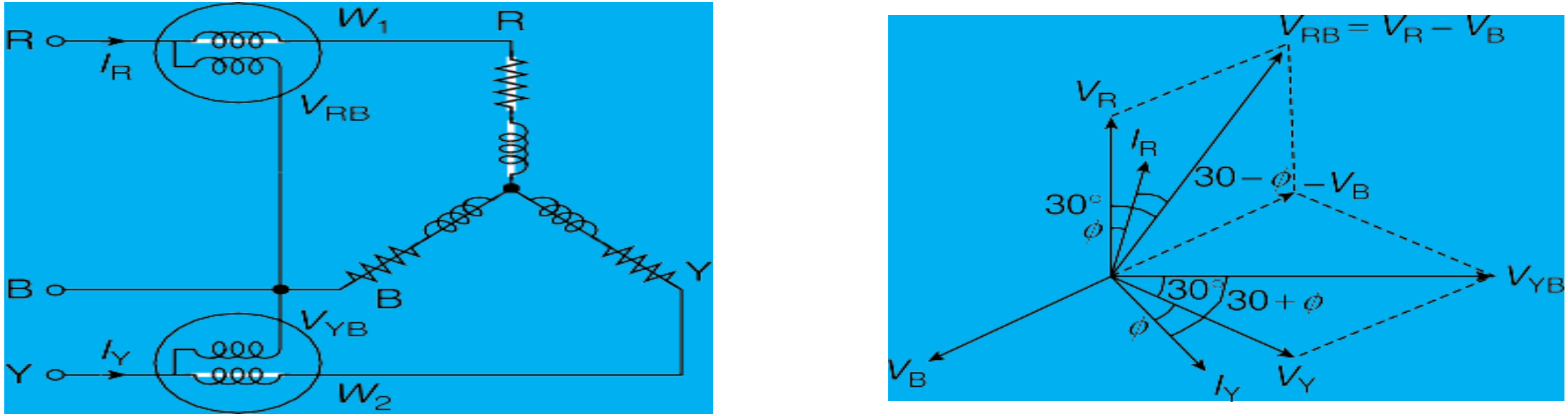


Fig.18 Measurement of Three-phase Power Using Two Single-phase Wattmeter's (a) Circuit Diagram and (b) Phaser Diagram

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

Let us add the two wattmeter readings, that is, W_1 and W_2.

$$W_1 + W_2 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30 - \phi) + \sqrt{3}\, V_{Ph} I_{Ph} \cos(30 + \phi)$$

$$= \sqrt{3} V_{Ph} I_{Ph} [\cos(30 - \phi) + \cos(30 + \phi)]$$

$$= \sqrt{3}\, V_{Ph} I_{Ph}\, 2 \cos\phi \cos 30^\circ \qquad = \sqrt{3}\, V_{Ph} I_{Ph}\, 2 \cos\phi \frac{\sqrt{3}}{2}$$

$$= 3 V_{Ph} I_{Ph} \cos\phi \quad \text{or} \quad W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \quad \text{----------equation (16)}$$

Thus, it is proved that the sum of the wattmeter readings is equal to the three-phase power. Now, when the two wattmeter readings are subtracted from each other, we obtain the following form:

$$W_1 - W_2 = \sqrt{3}\, V_{Ph} I_{Ph} [\cos(30^\circ - \phi) - \cos(30^\circ + \phi)$$

$$= \sqrt{3}\, V_{Ph} I_{Ph}\, 2 \sin\phi \sin 30^\circ$$

or $\sqrt{3}(W_1 - W_2) = 3 V_{Ph} I_{Ph} \sin\phi$ or $\sqrt{3}\,(W_1 - W_2) = 3 V_L I_L \sin\phi$ -------equation (17)

Dividing equation (17) by equation (16)

$$\frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3}\, V_L I_L \sin\phi}{\sqrt{3}\, V_L I_L \cos\phi} = \tan\phi$$

or $\phi = \tan^{-1} \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}$ Power factor $\cos\phi = \cos\tan^{-1} \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$

Thus, from the two wattmeter readings, we can calculate the total active and reactive powers and the power factor of the circuit.

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

Effect of Change in Power Factor a Wattmeter Readings:

We will now study the effect of change in load power factor on the wattmeter readings. Let us rewrite the wattmeter readings as follows:

$$W_1 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30 - \phi)$$

$$W_2 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30 + \phi)$$

We will consider a power factor of unity, 0.5, less than 0.5 and (0) and study the effect on the wattmeter readings,

1. At unity power factor, when cos ϕ= I, that is, ϕ=0 $W_1 = \sqrt{3}\, V_{Ph} I_{Ph} \cos 30^\circ$ $W_2 = \sqrt{3}\, V_{Ph} I_{Ph} \cos 30^\circ$

   Thus, at power factor =|, both the wattmeter readings will be positive and of equal value.

2. At 0.5 power factor, cos ϕ= 0.5, that is, ϕ= 60°

$$W_1 = \sqrt{3} V_{Ph} I_{Ph} \cos(-30^\circ) = \sqrt{3}\, V_{Ph} I_{Ph} \cos 30^\circ \qquad W_2 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30^\circ + 60^\circ) = 0$$

Thus, at power factor equal to 0.5, one of the wattmeter's will give zero reading.

3. When the power factor is less than 0.5, that is, ϕ> 60). Let us observe the wattmeter readings.

$$W_1 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30 - \phi)$$

$$W_2 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30 + \phi)$$

When ϕ> 60, W1 will give positive readings but W_2, will give a negative reading, Thus, for power factor less than 0.5, that is, for ϕ> 60°, one of the Watt meters will give a negative reading.

4. When the load is purely inductive or capacitive, the power factor will be zero, that is,

$$W_1 = V_L I_L \cos(30^\circ - 90^\circ) = V_L I_L \cos 60^\circ \qquad W_2 = V_L I_L \cos(30^\circ + 90^\circ) = -V_L I_L \sin 30^\circ$$

Both the wattmeter's show equal but opposite readings. Hence, the total power consumed will be zero.

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

8.3 Three-wattmeter Method:

In this method, three wattmeter's are used to measure three-phase power, Three wattmeter's are connected in each phase, as shown in Fig.19, and their pressure coils are connected between each phase and the neutral. This method is valid for the measurement of three-phase power for balanced and unbalanced loads. The main drawback of this method is the requirement of three wattmeter's.

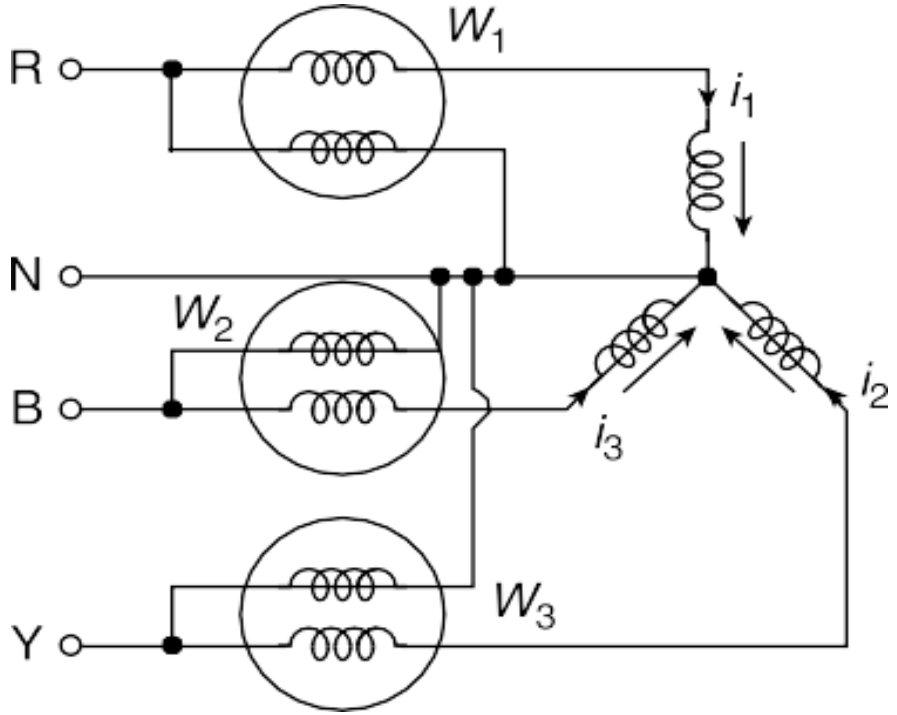


Fig.19 Measurement of three phase balanced or Unbalanced Power Using Three Single phase Watt meters

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

Example.11 In the two-wattmeter method of power measurement for a three-phase load, the readings of the wattmeter are 1OOO W and 550W. What is the power factor of the load?

Example.12 In the measurement of three-phase power by the two-wattmeter method, for a certain load, one of the watt meters reads 20kKW and the other 3kW after the current coil connection of one of the watt meters has been reversed. Calculate the power and power factor of the load.

Example.13 Draw the connection diagram for the measurement of power in a three-phase star-connected load using the two-wattmeter method. In one such a measurement, the load connected was 30 kW at 0.7 pf lagging. Find the reading of each wattmeter.

Example.14 A three-phase balanced load connected across a three-phase, 400V AC supply draws. a line current of 10.4. Two watt meters are used to measure the input power. The ratio of two wattmeter readings is 2:1. Find the readings of the two watt meters.

Example.15 Three equal impedances, each consisting of R and L in series are connected instar and are supplied from a 400) V. 30 Hz, three-phase, three-wire balanced supply system, The power input to the load is measured by the two-wattmeter method and the two watt meters read 3kW and 1 Kw respectively. Determine the values of R and L connected in each phase.

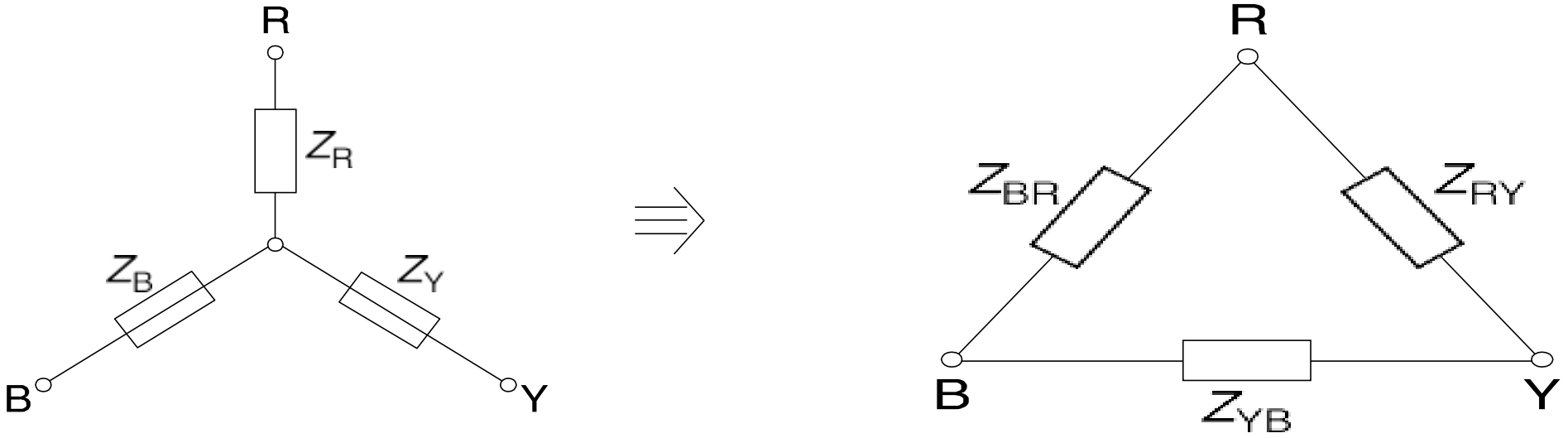
Example.16 The power input to a three-phase motor is measured by two single-phase watt meters. The total input power has been measured as equal to 15kW and the power factor calculated as 0.5, What have been the readings of the two watt meters?

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

8.4 Star to Delta and Delta to Star Transformation

Load impedances may often be connected in either star or in delta. It may be necessary to convert star connected impedances to equivalent delta or delta connected impedances into equivalent star for the purpose of circuit analysis. The formulae for such conversion are given as follows.

Conversion of Star Load into Delta Load



The equivalent delta impedances in terms of star impedances across the three terminals as shown in Figure are as follows:

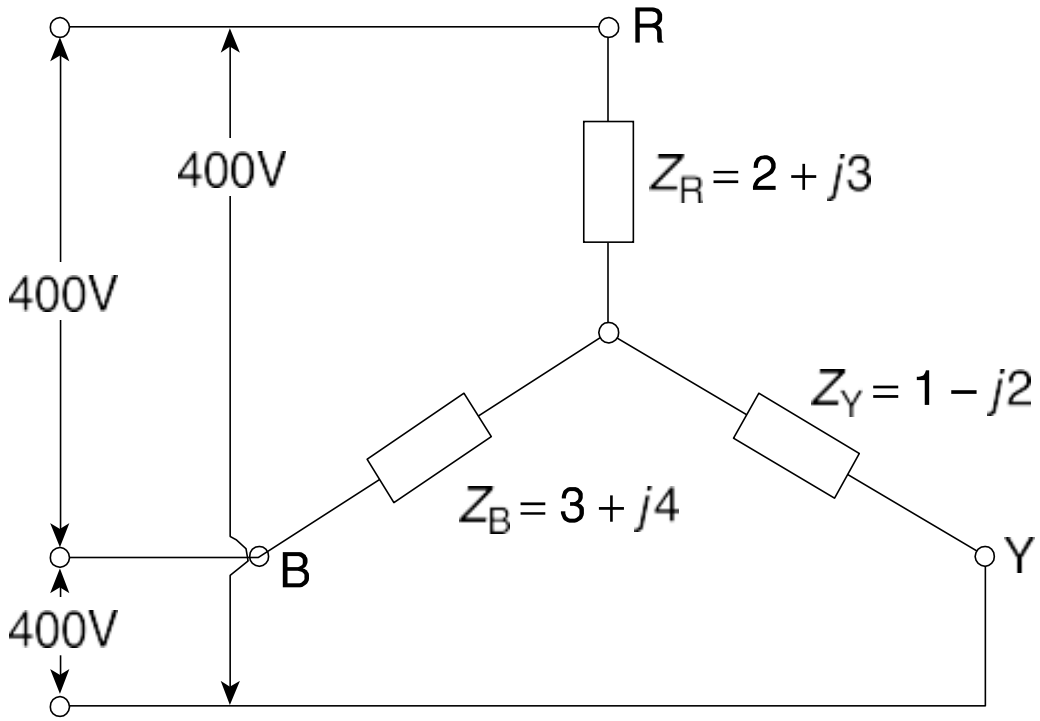
$$Z_{RY} = Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}$$

$$Z_{YB} = Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R}$$

$$Z_{BR} = Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}$$

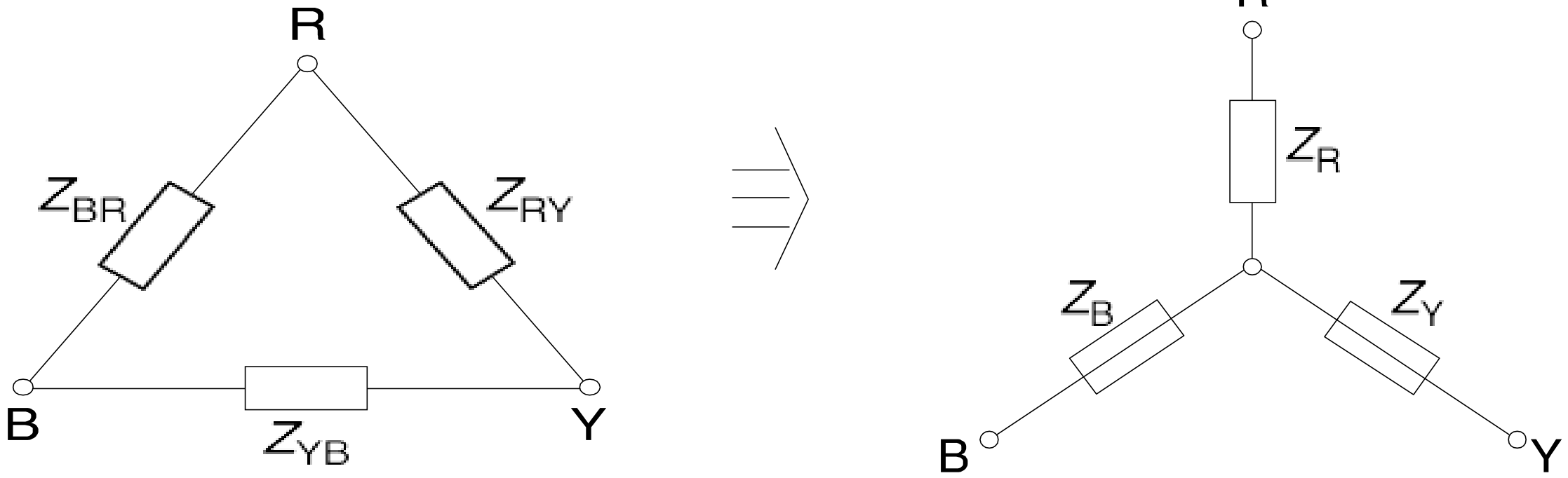
Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

Example. 17 A three-phase 400, 50 HZ supply is connected across a three-phase load as shown in Figure . Calculate the equivalent delta load.



Conversion of Delta Load into Star Load :

Load impedances may also connect in delta for the conversion of delta connected impedances into equivalent star connected impedances, as shown in Figure 21, the following relations will hold good.



Delta to Star Transformation

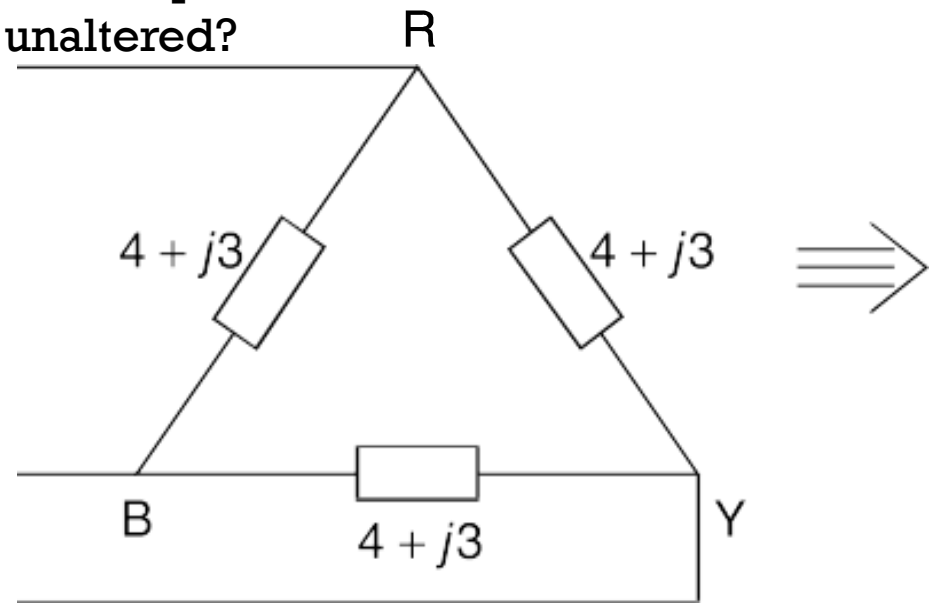
$$Z_R = \frac{Z_{RY} Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

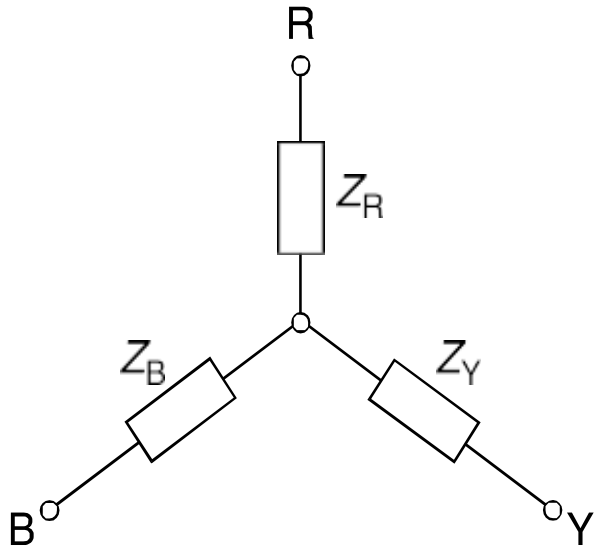
$$Z_Y = \frac{Z_{RY} Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$Z_B = \frac{Z_{YB} Z_{BR}}{Z_{BR} + Z_{YB} + Z_{BR}}$$

Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

Example18. A 440V.,50 Hz, three-phase supply is connected across a delta-connected load having identical impedance of
4 + j3 Ωfor each phase. What should be the values of equivalent star impedances so as to keep the line currents unaltered?

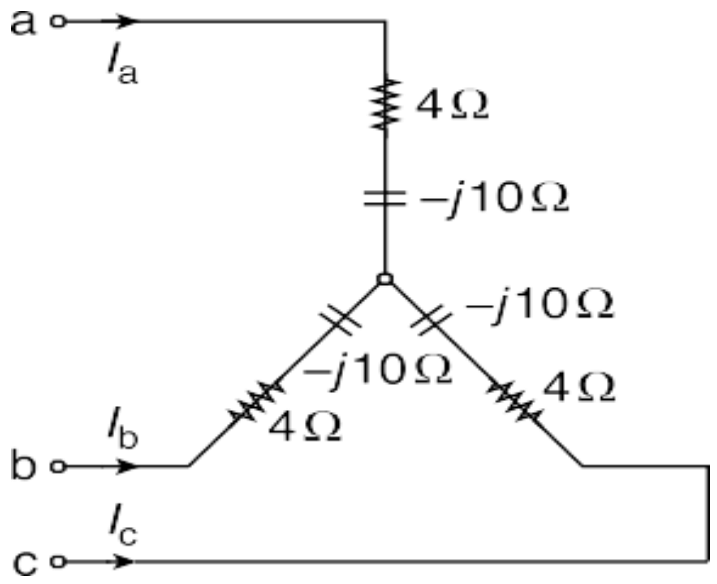




Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

9. NUMERICALS BASED ON THREE-PHASE BALANCED LOAD

- Example 19 A. Star-connected Three-phase Load Has A Resistance Of 4Ω And A Capacitive Reactance Of 100Ω In Each Phase. If It Is Fed From A 400V Three-phase Balanced Supply, Find the Line Current, Total Volt Amperes, Active And Reactive Power.



Reference: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

10. METHOD OF SOLVING PROBLEMS ON UNBALANCED LOAD

- In Reality. It is Difficult To Maintain A Perfect Balanced Three-phase Load On A Power System, Unbalance Load Condition Occurs Due To Uneven Loading Of The Three Phases, Causing Either Unequal Magnitude Of The Three-phase Impedances Or The Impedance Angles. Unbalance Load Condition Will Occur Due To Unbalanced Voltage Supply Also. Below is the Example Of Unbalanced Load.
- **Example 20.** A 400V Three-phase Four-wire System Has Loads Of 4 —J7, 3 —J4 ,4 + J5 Ωconnected In Star. Find The Line Currents, Neutral Current And Total Power,